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ASYMMETRIC IMPACT OF A JET WITH AN IDEAL NONCOMPRESSIBLE LIQUID

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UDC 532.522

The problem of impact of a jet belongs among classical problems. The question of asymmetric impact of a jet is of special importance in view of the development of new methods for treating metals by means of explosive energy and, in the first instance, welding by explosion [1, 2].

We consider steady-state flow as a result of the impact of two planar jets A_1 and A_2 having at infinity prescribed thicknesses h_0 and H_0 and velocity direction v (Fig. 1). It is necessary to determine the parameters of the two jets B_1 and B_2 formed. Concerning values of velocity v at infinity, then in the case of streams of identical density the velocities should be identical for all four jets. This follows from the fact that free lines of the flow A_1B_1, B_1A_2, A_2B_2, B_2A_1 are lines of constant velocities. We shall consider flow with one critical point O at which we place the origin of a Cartesian coordinate system, and we set axis x parallel and toward the velocity of the approach stream in jet A_1. The angle between converging jets A_1 and A_2 is designated in terms of α, and between diverging jets B_1, B_2, and axis x in terms of φ and ψ, respectively (see Fig. 1). Let the thickness of jet B_1 be h and that of jet B_2 be H. Then if it is assumed that h_0, H_0, and α are given, in order to determine the remaining four unknowns h, H, φ, and ψ we have three equations in all following from the laws of conservation for mass and flow, and impact for an ideal noncompressible liquid:

$$h + H = h_0 + H_0; \tag{1}$$

$$H \cos \psi - h \cos \varphi = h_0 + H_0 \cos \alpha; \tag{2}$$

$$H \sin \psi - h \sin \varphi = H_0 \sin \alpha. \tag{3}$$

Relationships (2) and (3) are projections of the stream impact on the axis of coordinates x and y, respectively. Thus, the problem appears to be indeterminate. An attempt to make it definite by introducing a supplementary arbitrary hypothesis belongs to Platini [3]. It was suggested that straight line B_2 and return line B_1 of the diverging jet at infinity move in opposite directions

$$\overline{\varphi} = \overline{\psi}. \tag{4}$$

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There is another possibility to close the sets of equations (1)-(3) if symmetrical impact is considered, i.e., impact of two jets of identical thickness $h_0 = H_0$. Then for a diverging jet, as a result of symmetry the whole picture of flow is

$$\varphi = \psi = \alpha/2. \tag{5}$$

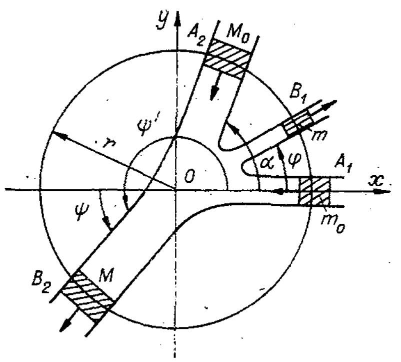

Fig. 1

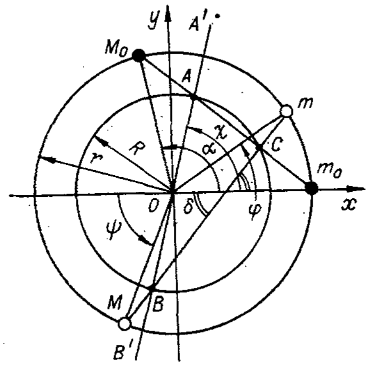

Fig. 2

The case of symmetrical impact was suggested by Lavrentiev on the basis of hydrodynamic theory for accumulation and armor piercing [4]. In order to obtain additional information about the flow of an impacting jet so that the set of equations (1)-(3) can be closed, we determine the position of the mass centers of inertia for specially isolated volumes of liquid in the impinging diverging jets. At a sufficiently large distance r ($r \to \infty$) from point to impact O mass flow in unit time in converging jets A_1 and A_2 is $m_0 = h_0 v$ and $M_0 = H_0 v$. Center of inertia A (Fig. 2) for these separate masses of liquid is determined by the radius vector

$$\mathbf{R}_0 = \frac{m_0 \mathbf{r}_{01} + M_0 \mathbf{r}_{02}}{m_0 + M_0},$$

where r_{01} and r_{02} are radius-vectors for masses m_0 and M_0.

Similarly for divergence of jets B_1 and B_2, the radius-vector of the center of inertia (point B) for mass flows in unit time occurring across their transverse section at distance r from point O:

$$\mathbf{R} = \frac{m\mathbf{r}_1 + M\mathbf{r}_2}{m + M}.$$

Here $m = hv$, $M = Hv$, and r_1 and r_2 are their radius-vectors, respectively. We calculate the projected radius-vectors R_0 and R on the coordinate axes. Projections on axis x are determined by the expressions

$$\begin{aligned} R_{0x} &= \frac{m_0}{m_0 + M_0} r_{01} + \frac{M_0}{m_0 + M_0} r_{02} \cos\alpha, \\ R_x &= \frac{m}{m + M} r_1 \cos\varphi - \frac{M}{m + M} r_2 \cos\psi, \end{aligned} \tag{6}$$

and on axis y by

$$R_{0y} = \frac{M_0}{m_0 + M_0} r_2 \sin\alpha, \quad R_y = \frac{m}{m + M} r_1 \sin\varphi - \frac{M}{m + M} r_2 \sin\psi. \tag{7}$$

Bearing in mind apparent equalities $r_{01} = r_{02} = r_1 = r_2$ and $m_0 + M_0 = m + M$, and comparing relationship (6) with (2) and relationship (7) with (3), we find that in order to fulfill the conservation rule for flow impact (2) and (3) it is necessary that

$$R_{0x} = -R_x, \; R_{0y} = -R_y,$$

i.e., the radius-vectors of centers of inertia R_0 and R separated for masses m_0, M_0, and m, M should be equal in absolute value and directed in the opposite directions

$$\mathbf{R}_0 = -\mathbf{R}.$$

Thus, radius-vectors R_0 and R lie ion one straight line A'B' passing through the starting coordinate. From relationships (6) and (7) it is easy to find the slope angle ψ of straight line A'B' to axis x

$$\operatorname{tg}\chi=\frac{R_{0y}}{R_{0x}}=\frac{M_0\sin\alpha}{m_0+M_0\cos\alpha}=\frac{H_0\sin\alpha}{h_0+H_0\cos\alpha} \tag{8}$$

and the ratio of radii for circles on which lie separate masses of liquid in converging jets and their center of inertia:

$$n=\frac{R}{r}=\frac{\sqrt{R_{0x}^2+R_{0y}^2}}{r}=\frac{\sqrt{m_0^2+M_0^2+2m_0M_0\cos\alpha}}{m_0+M_0}. \tag{9}$$

We break down the case presented in Fig. 2 when $M_0 > m_0$. It is evident that point A (center of inertia for masses m_0 and M_0) is a common point of circle R and straight lines A'B' and m_0M_0. If $M_0 < m_0$, then straight line A'B' should pass through a second point C of intersection of straight line m_0M_0 with circle r. Since point B is the center of inertia of masses m and M, then points B, m, and M lie on a single straight line. We refer all linear values to circle radius r. We consider a cluster of straight lines passing through point $B\{-n\cos\chi, -n\sin\chi\}$ and lying within central angle α formed by converging jets A_1 and A_2. The equation for the cluster of straight lines is:

$$y=\operatorname{tg}\delta\cdot(x+n\cos\chi)-n\sin\chi, \tag{10}$$

where δ is the slope angle of straight lines to the positive direction of axis x. Intersection of these straight lines with a circle $x^2 + y^2 = 1$ makes it possible to determine the coordinates of points m and M lying on circle r = 1:

$$\begin{aligned} x_1&=\cos\varphi=n\sin\delta\cdot\sin(\chi-\delta)+\cos\delta\cdot\sqrt{1-n^2\sin^2(\chi-\delta)},\\ x_2&=-\cos\psi'=\cos\psi=-n\sin\delta\cdot\sin(\chi-\delta)+\\ &\quad+\cos\delta\cdot\sqrt{1-n^2\sin^2(\chi-\delta)}. \end{aligned} \tag{11}$$

Since relationship (11) should also describe the case of symmetrical impact of a jet $m_0 = M_0$, then by substituting in (11) condition (5) for symmetrical impact $\varphi = \psi = \alpha/2$ and having from (8) $\chi = \alpha/2$ and from (9) $n = \cos(\alpha/2)$, we obtain $\delta = \alpha/2$. Consequently, the solution of the stated problem is

$$\cos\varphi=n\sin\frac{\alpha}{2}\cdot\sin\left(\chi-\frac{\alpha}{2}\right)+\cos\frac{\alpha}{2}\sqrt{1-n^2\sin^2\left(\chi-\frac{\alpha}{2}\right)}; \tag{12}$$

$$\cos\psi=-n\sin\frac{\alpha}{2}\cdot\sin\left(\chi-\frac{\alpha}{2}\right)+\cos\frac{\alpha}{2}\sqrt{1-n^2\sin^2\left(\chi-\frac{\alpha}{2}\right)}. \tag{13}$$

By noting that $\sin\left(\chi-\frac{\alpha}{2}\right)=\frac{M_0-m_0}{M_0+m_0}\sin\frac{\alpha}{2}$, it is possible to simplify these relationships, reducing them to the form

$$\cos\varphi=\mu\sin^2\frac{\alpha}{2}+\cos\frac{\alpha}{2}\sqrt{1-\mu^2\sin^2\frac{\alpha}{2}}; \tag{14}$$

$$\cos\psi=-\mu\sin^2\frac{\alpha}{2}+\cos\frac{\alpha}{2}\sqrt{1-\mu^2\sin^2\frac{\alpha}{2}}, \tag{15}$$

where

$$\mu=\frac{M_0-m_0}{M_0+m_0}=\frac{H_0-h_0}{H_0+h_0}.$$

From (1)-(3) and (14), (15), we determine thicknesses h and H of diverging jets B_1 and B_2

$$h=\frac{h_0+H_0}{2}\left(1-\frac{\cos\frac{\alpha}{2}}{\sqrt{1-\mu^2\sin^2\frac{\alpha}{2}}}\right); \tag{16}$$

$$H=\frac{h_0+H_0}{2}\left(1+\frac{\cos\frac{\alpha}{2}}{\sqrt{1-\mu^2\sin^2\frac{\alpha}{2}}}\right). \tag{17}$$

For symmetrical impact with $h_0 = H_0$ ($\mu = 0$) from (14) and (15) we obtain Eq. (5), and from (16) and (17) we obtain

$$h_* = h_0\left(1-\cos\frac{\alpha}{2}\right); \tag{18}$$

$$H_* = h_0\left(1+\cos\frac{\alpha}{2}\right). \tag{19}$$

Thus, Eqs. (14)-(17) for symmetrical impact give relationships (18) and (19) between parameters for an impacting jet obtained previously in hydrodynamic theory for accumulation [4] for thicknesses of a cumulative jet h_* and stamp H_*. It is interesting to note that from (12) and (13) or (14) and (15) for frontal impact of a converging jet with $\alpha = \pi$ we have

$$\cos\varphi=\mu,\quad \cos\psi=-\mu,\quad h=H=\frac{h_0+H_0}{2}.$$

This means that diverging streams are uniform in thickness and symmetrical relative to axis x. The same result follows from hydrodynamic accumulation theory [4] with turning of a given flow when jets of identical thickness collide at an angle.

Calculated results for certain cases of asymmetric impact of a jet are shown in Table 1, where values are not given for angles ψ which are formed by diverging jet B_2 with axis x. This is not by accident. We refer to Fig. 2. It is easy to demonstrate by simple calculation that straight line Mm from cluster of straight lines (10) that passes through point B, for which the slope angle to the positive direction of axis x equals $\alpha/2$, should pass through point C of the intersection of straight line m_0M_0 with a circle of radius R. Then for this circle the angle ABC described has a bearing on the arc of the circle AC, on which the central angle AOC also has a bearing. Consequently, $\angle ABC = 1/2\angle AOC = \chi - \alpha/2$. On the other hand, from isosceles triangle mOM it is found that angle OMm equals $(1/2)(\psi - \varphi)$. Since angle ABC is an external angle of triangle MOB, then we have the equality

$$\chi-\alpha/2=(\chi-\psi)+(1/2)(\psi-\varphi),$$

whence

$$\varphi+\psi=\alpha. \tag{20}$$

Thus, angles φ and ψ are connected by relationship (20). At the same time, (20) indicates the groundlessness of the Platini hypothesis $\varphi = \psi$. In addition, (20) is in fact the missing equation necessary to close the set of equations (1)-(3). By solving the set of equations (1)-(3) and (20) we arrive at the same expressions (12)-(15) for determining h, H, φ, and ψ, which are found in analyzing the position of centers of inertia for isolated elements in converging and diverging jets.

TABLE 1

μ	M_0/m_0	φ, deg							
		α, deg							
		30	45	60	90	120	150	170	180
0	1	15	22,5	30	45	60	75	85	90
0,2	1,5	12,0	18,1	24,3	36,9	50,0	63,9	73,5	78,5
0,333333	2	10,1	15,2	20,4	31,4	43,2	56,2	65,6	70,5
0,5	3	7,6	11,5	15,5	24,3	34,4	46,1	55,1	60,0
0,6	4	6,1	9,2	12,5	19,9	28,7	39,6	48,3	53,1
0,818182	10	2,8	4,3	5,9	9,7	14,9	22,8	30,4	35,1
0,980198	100	0,3	0,5	0,7	1,1	1,9	3,8	7,5	11,4

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ASYMPTOTIC EXPANSIONS OF THIN AXISYMMETRIC CAVITIES

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The theory of flow around a thin body [1] enables one to obtain expansions for the potential of the velocity field in terms of a small parameter χ (thickness of the body) with any degree of accuracy. The first six terms of this expansion have the following orders of magnitude: 1, $\chi^2 \ln \chi$, χ^2, $\chi^4 \ln^2 \chi$, $\chi^4 \ln \chi$, χ^4. In most works on cavitating flows, calculations are carried out by using only the two first terms of the expansion, e.g., [2]. The problem of determining the free boundary reduces, in that approximation, to solution of an ordinary differential equation. For practical reasons one should take into account also the third term of the expansion together with the second, which is of the order close to χ^2, while the subsequent three terms of the expansion are of essentially smaller, close to χ^4, order. In presence of the term χ^2, the potential of the flow is expressed by the integral operator acting on the function defining the boundary of the body placed in the stream [1]. Therefore, the equation of the free boundary is a nonlinear integrodifferential equation. It seems that only [3] contains calculations in this approximation. The solution of the integrodifferential equation is shown in the form of an expansion in negative powers of $\ln \chi$. In this work the Riabushinskii scheme is used in order to obtain an asymptotic expansion for the drag force F in powers of a small parameter ε_1 for arbitrary thickness of the cavitating body. The first term of this expansion agrees with the asymptotic formula given in [4]. For the flow in the Kirchhoff scheme ($\sigma = 0$) an expansion is obtained for $x \to \infty$ for the free-jet boundary. Its asymptotic behavior agrees with the law of jet expansion obtained independently by Gurevich and Levinson [5].

1. Theory of Nonseparating Flow around a Thin Body

Here we consider the problem of flow around a thin body of rotation by a stationary stream of nonviscous incompressible fluid. Let all lengths be referred to the half-length of the body ℓ_x, velocities be referred to the velocity of the incoming stream at infinity v_∞, and the boundary of the body in the meridional plane be defined by the equation

$$y = \chi f(x), \ -1 \leqslant x \leqslant 1. \tag{1.1}$$

The small parameter $\chi \ll 1$ is a measure of the relative thickness of the body whose shape is given by the function f(x). The potential Φ of the velocity field is to be found from the solution of the boundary-value problem

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{1}{y}\frac{\partial\Phi}{\partial y} = 0, \tag{1.2}$$
$$\frac{\partial\Phi}{\partial y} \Big/ \frac{\partial\Phi}{\partial x} = \chi\frac{df}{dx}, \quad |x| \leqslant 1, \quad y = \chi f(x),$$
$$\Phi \to x, \ x^2 + y^2 \to \infty.$$

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